INFRARED QCD AND THE RENORMALISATION GROUP*

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We study the infrared regime of QCD by means of a Wilsonian renormalisation group. We explain how, in general, the infrared structure of Green functions is deduced in this approach. Our reasoning is put to work in Landau gauge QCD, where the leading infrared terms of the propagators are computed. The results support the Kugo-Ojima scenario of confinement. Possible extensions are indicated.

1. Introduction

Many aspects of QCD are well understood in its ultraviolet limit where the gauge coupling is small as a consequence of asymptotic freedom. Therefore, perturbation theory is expected to provide a viable description of QCD phenomena at high energies or high temperature. On the other hand, in the infrared limit, quarks and gluons are confined to hadronic states and the gauge coupling is expected to grow large. In consequence, a reliable description of low energy phenomena -like confinement, the physics of bound states, chiral symmetry breaking or the confinement-deconfinement transition- call for a non-perturbative analysis.

Renormalisation group methods provide important analytical tools in the study of non-perturbative phenomena. In this contribution, we study Landau gauge QCD in the non-perturbative infrared regime [1] by means of an exact (or functional) renormalisation group [2,3]. The strength of the

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approach is its flexibility when it comes to approximations. Furthermore, efficient optimisation procedures are available, increasing the domain of validity and the convergence of the flow.[4] We discuss how, in general, the momentum structure of Green functions in the infrared is deduced from non-perturbative renormalisation group equations. As an application we compute the leading non-perturbative infrared coefficients (or anomalous dimensions) for the gluon and ghost propagator in Landau gauge. Our results are discussed in the light of the Kugo-Ojima confinement criterion and earlier findings based on other non-perturbative methods. For technical details we refer to the original publication.[1]

2. Renormalisation group for QCD

The exact renormalisation group is based on the Wilsonian idea of integrating-out momentum degrees of freedom within a path integral representation of quantum field theory. Central to this approach is the effective action Γ_k, where quantum fluctuations with momenta $q^2 > k^2$ are already integrated out. The renormalisation group equation for Γ_k is given by

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R}\,\partial_t R\,. \tag{1}$$

Here, $t = \ln k$ is the logarithmic scale parameter, Tr denotes a trace over loop momenta q and a sum over indices and fields, and $R(q^2)$ is the infrared momentum cutoff at momentum scale k. R obeys a few restrictions which ensure that the flow is well-defined and finite both in the ultraviolet and the infrared. The flow interpolates between the initial (classical) action in the ultraviolet and the full quantum effective action in the infrared where the cutoff is removed. For gauge theories, the flow is amended by a set of modified Slavnov-Taylor identities.[5,6] They ensure the requirements of gauge symmetry for Green functions in the physical limit $k \to 0$. Since this approach allows even for truncations which are non-local in momenta and the fields it is particularly useful for gauge theories. Furthermore, it is worth emphasising that the flow only involves fully dressed propagators and vertices. Therefore, the correct RG scaling properties are represented by the full flow and truncations with the correct symmetry properties.

So far the flow equation (1) has been applied to Landau gauge QCD for a determination of the heavy quark effective potential and effective quark interactions above the confinement scale.[7] For an implementation in axial gauges and further applications in Yang-Mills theories see Refs. [8].

3. Infrared regime of QCD

In quantum field theory, important physical information for low energy phenomena is given through the momentum structure of Green functions in the deep infrared regime. In QCD, the characteristic scale differentiating between strong and weak coupling is $\Lambda_{\rm QCD} \approx 200$ MeV. The strongly coupled deep infrared regime is defined as

$$p^2 \ll \Lambda_{\rm QCD}^2 \tag{2}$$

and p denotes a momentum argument of a QCD Green function. As a consequence of confinement, it is expected that gluon and ghost propagators display strong deviations from a simple particle pole for sufficiently small momenta. Within covariant linear gauges, the necessary conditions for confinement in terms of local fields were formulated by Kugo and Ojima.[9] In Landau gauge, they state that the gluonic correlations are suppressed in the infrared as a consequence of a mass gap, while the ghost correlations are infrared enhanced and dominant. This type of behaviour has already been detected in solutions of truncated Schwinger-Dyson equations [10] and stochastic quantisation [11], and lattice simulations [12].

Now we turn to the infrared analysis based on (1), where two-point functions and propagators depend additionally on the cutoff scale k. As long as $k^2 \gg p^2$, the propagators barely differ from the classical ones, given that no quantum fluctuations with momenta p or smaller have yet been integrated out. In turn, as soon as

$$k^2 \ll p^2 \tag{3}$$

all quantum fluctuations have been integrated out and physical quantities as well as general vertex functions upon appropriate rescaling, are no longer affected by the infrared cutoff. Consequently, the non-trivial infrared behaviour of physical Green functions resides in the momentum regime given by (2) and (3). It can be shown that Green functions in the regime (2) depend on the cutoff scale k only parametrically through dimensionless ratios, or through k-dependent renormalisation group factors that leave the full action invariant.[1] This property implies a fixed point behaviour and strongly facilitates the evaluation of (1) in given truncations.

In order to integrate the flow (1) in the infrared regime, we introduce an appropriate truncation of Γ_k and retain the full momentum dependence of the QCD propagators. The truncation is amended by vertices which fulfil the truncated Slavnov-Taylor identities. The validity of the truncation has recently been confirmed on the lattice.[13] The gluon and ghost two-point

function are parametrised as

$$\Gamma^{(2)}_{k,A/C}(p^2) = z_{A/C} \cdot Z_{A/C}(x) \cdot p^2 \,. \tag{4}$$

Here, $x = p^2/k^2$, z denotes a possibly k-dependent RG factor, and $Z(x) = x^\kappa(1+\delta Z(x))$ parametrises the non-trivial modifications of the momentum structure through quantum fluctuations. The indices A/C refer to the gluon/ghost fields, respectively. In (4), we have suppressed the trivial colour structure and the transversal projector for the gluons. The longitudinal modes do not contribute to the flow in Landau gauge. Note that (4) is the most general parametrisation of the propagators and no assumptions have been made upon their structure. In the deep infrared region the behaviour of (4) is dominated by the term x^κ, where $\kappa_{A/C}$ is a non-perturbative anomalous dimension of the gluon and ghost propagator in Landau gauge. The function δZ constitutes the transition behaviour between the deep infrared regime and the cutoff regime. In addition, non-renormalisation of the ghost-gluon vertex implies [14]

$$\kappa_A = -2\kappa_C \,, \qquad \alpha_s = \frac{g^2}{4\pi}\frac{1}{z_A z_C^2} \,. \tag{5}$$

The coefficients $\kappa_{A/C}$ and the value for α_s in the deep infrared regime are deduced from integrating the flow for the propagators using (2) and (3). This leads to two integral equations for $\delta Z_{A/C}$ of the form

$$\delta Z_{A/C}(x) = F_{A/C}[\delta Z_{A/C}, \kappa_{A/C}, \alpha_s] \,. \tag{6}$$

Explicit expressions for the integrals F are given in Ref. [1]. The simultaneous solutions of (6) lead to explicit solutions for the infrared coefficients κ, α_s and δZ. To leading order, the back-coupling of δZ on the right hand side of (6) can be neglected. In this limit, we find

$$\kappa_C = 0.59535\cdots \qquad \alpha_s = 2.9717\cdots \tag{7}$$

independently of the cutoff function R. Furthermore, the numerical values agree with the most advanced results obtained within the Dyson-Schwinger approach and stochastic quantisation. Iterating (6) beyond leading order ($\delta Z \neq 0$), we find that κ_C varies slightly with the cutoff, ranging between 0.539 for the sharp cutoff and (7) for appropriately optimised ones. More recently, the infrared region has also been studied in Ref. [15].

4. Discussion

We have shown how, in general, the momentum dependence of Green functions is extracted in the infrared regime based on an exact renormalisation

group. Applied to QCD in the Landau gauge, we detected a strong enhancement of the ghost propagator while the gluon propagator develops a mass gap. Our results provide a further independent evidence for the Kugo-Ojima scenario of confinement. In the simplest possible truncation with dressed propagators our results match the state-of-the-art within both the Dyson-Schwinger and the stochastic quantisation approach. This is quite remarkable, in particular in view of the conceptual and technical differences between these methods. Current work deals with straightforward extensions of the present analysis and covers vertex corrections, investigations of QCD Green functions at finite temperature, and dynamical quarks. In either case, the correct RG scaling as well as the inherent finiteness of the integrated flow are most important.

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